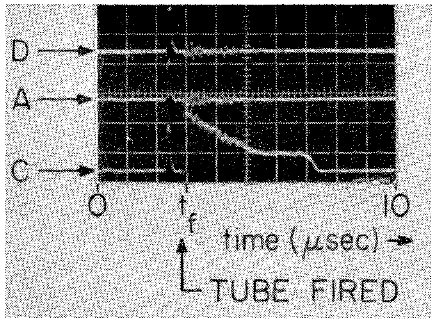

Fig. 5—Oscillogram of detected RF envelopes. D) The upper trace oscillographs the spike which is allowed to pass through the switch tube, demonstrating the hold-off capability of the switch. The spike peak power level is 130 kw. A) The center trace depicts the power received by the pick-up horn. The flat leakage is about 35–40 db below the peak power level of the spike. The spike peak power is approximately 140 kw. The flat leakage cannot be seen on this trace but reference may be made to Fig. 2. C) The lower trace is the desired waveform. The small spike appearing at the left is due to the reflections of the switch when cold (VSWR ≈1.07) and has a peak amplitude of approximately 80 w. The important portion follows after the switch has been fired. It can be seen that the waveform attenuates slowly with increasing time illustrating an increasing concentration of charge density in the plasma. The attenuation remains constant in the final portion of the pulse marking the onset of equilibrium conditions in the plasma. This latter part of the pulse has a power level of approximately 40 w. These oscillograms yield no information on the afterglow.

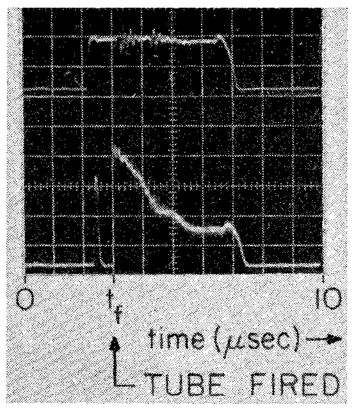

Fig. 6—Oscillogram of detected envelopes. The upper waveform is the detected envelope of the signal irradiating the bottle and creating the plasma. The pulse is 5 μsec in width, occurs at a PRR of 60 pps, has a peak power level of 230 kw and a frequency of 5.5 Gc. The lower trace is the detected RF envelope at port C of the circulator. The instantaneous attenuation of the lower trace relative to the upper trace, now easily available, is of critical importance in plasma diagnosis.

travels through the switch exhibiting the hold-off capability of the device. The switch may be fired at any time during the RF pulse. This allows a useful flexibility in the event that the spike width varies. The flat portion of the transmitted pulse is reflected when the tube fires. Switch losses are constant during the switching time and are known. The tube remains on for 30 μsec. This flat portion, when displayed on a CRO, yields the useful information. The attenuation of the flat portion relative to the irradiating pulse power at one instant of time is found by noting the over-all losses of the system including the plasma. The system is then calibrated by removing the gas from the bottle and measuring the attenuation at the same instant of time. With the system calibrated, the CRO then displays db attenuation against time over the entire flat portion. This information is then related to the electron concentration. Fig. 6 depicts the flat portion of the transmitted pulse displayed directly under the irradiating pulse. Explanations are given in the caption.

Conclusion

The use of the fast acting (20–30 nsec) high-peak-power (~1 Mw) switch in conjunction with a three-port circulator has allowed us to 1) separate, in a clean manner, the spike of energy transmitted through a plasma bottle during the formative time lag from the flat low level (~35–40 db below the spike) portion and 2) display the flat portion superimposed under the irradiating pulse in order to obtain instantaneous attenuation against time. The component which is necessary to do this is described by Goldie.[1] The switch electrical characteristics may also be found by referring to Goldie.[1]

The authors would like to thank Prof. J. Shmoys for his advice and encouragement.

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Dominant Mode Propagation in Ge and Si with Carrier Density Decaying Exponentially in Time*

Nag and Das[1] have recently made a theoretical study of microwave propagation in a semiconductor-filled rectangular waveguide when the semiconductor has a time dependent carrier density. They have assumed ϵ and σ to be time dependent in the wave equation for the TE_{01} mode wave and obtained a solution for the electric field E_x by perturbation techniques for small changes in carrier density. An equivalent propagation constant can be obtained for Germanium and Silicon by solving the wave equation for E_x, assuming no time variations in ϵ and σ, and then later inserting their time dependence. This is an implicit physical assumption in an earlier work of Jacobs, *et al.*[2]

Consider the case of X band propagation or $\omega = 2\pi \times 10^{10}$ cps in Ge or Si having a lifetime $\tau_c = 10^{-6}$ sec. In one microwave period of duration, $T = 10^{-10}$ sec, the fractional change in the *excess carrier* conductivity is

$$\frac{\sigma_1\left[e^{-t/\tau_c} - e^{-(t+T)/\tau_c}\right]}{\sigma_1 e^{-t/\tau_c}} \cong \frac{T}{\tau_c} = 10^{-4}. \quad (1)$$

The fractional change in the total conductivity would be even smaller. For longer lifetimes[2,3] and higher frequencies, T/τ_c would be smaller than the value obtained above. If we define the propagation constant in a semiconductor-filled guide as $j\Gamma$, then Γ is given by

$$\Gamma = \left[\omega^2\mu\epsilon - (\pi/a)^2 - j\omega\mu\sigma\right]^{1/2}. \quad (2)$$

Using the time dependence for σ and ϵ as given by (3) of Nag and Das,[1] (2) may be written as

$$\Gamma = \left[\omega^2\mu\{\epsilon_s - \epsilon_1 e^{-t/\tau_c}\} - (\pi/a)^2 - j\omega\mu\{\sigma_s + \sigma_1 e^{-t/\tau_c}\}\right]^{1/2} \quad (3)$$

or

$$\Gamma = \gamma_s\left[1 - \frac{\omega^2\mu\epsilon_1 + j\omega\mu\sigma_1}{\gamma_s^2}\cdot e^{-t/\tau_c}\right]^{1/2} \quad (4)$$

where

$$\gamma_s = \left[\omega^2\mu\epsilon_s - (\pi/a)^2 - j\omega\mu\sigma_s\right]^{1/2}.$$

For small changes in carrier density, (4) can be approximated by

$$\Gamma = \gamma_s\left(1 - \frac{\omega^2\mu\epsilon_1 + j\omega\mu\sigma_1}{2\gamma_s^2}\cdot e^{-t/\tau_c}\right). \quad (5)$$

For Ge and Si at X band frequencies or higher, the time dependent propagation constant obtained by Nag and Das[1] can be shown to be equivalent to that of (5). From their work, Γ can be written as

$$\Gamma = \gamma_s(1 + j\alpha_2 e^{-t/\tau_c}) \quad (6)$$

where $j\alpha_2$ can be written as[4]

$$j\alpha_2 = -\frac{\mu}{2\gamma_s^2}\left[\omega^2\epsilon_1\left\{1 - \frac{1}{\omega^2\tau_c^2}\left(1 + \frac{\sigma_1\tau_c}{\epsilon_1}\right)\right\} + j\omega\sigma_1\left\{1 + \frac{2\epsilon_1}{\sigma_1\tau_c}\right\}\right]. \quad (7)$$

For Ge and Si, the values of the terms in the brackets {} both tend to unity. For $\omega = 2\pi \times 10^{10}$ cps and $\tau_c = 10^{-6}$ sec, $\omega^2\tau_c^2 \cong 4 \times 10^9$. The ratio of ϵ_1 to σ_1 is given by[5]

$$\frac{\epsilon_1}{\sigma_1} = \frac{\left\langle \dfrac{\tau^2}{1+\omega^2\tau^2} \right\rangle}{\left\langle \dfrac{\tau}{1+\omega^2\tau^2} \right\rangle} \quad (8)$$

where τ is an energy dependent relaxation time and the brackets ⟨⟩ denote weighted averages over energy. For a constant relaxation time, $\epsilon_1/\sigma_1 = \tau$. A reasonable room temperature estimate of τ is 10^{-13} sec; thus, we choose $\epsilon_1/\sigma_1 = 10^{-13}$ sec. Then it is readily seen that

$$\frac{1}{\omega^2\tau_c^2}\left(1 + \frac{\sigma_1\tau_c}{\epsilon_1}\right) \cong 2.5 \times 10^{-3}, \quad (9a)$$

$$\frac{2\epsilon_1}{\sigma_1\tau_c} \cong 2 \times 10^{-7}. \quad (9b)$$

For higher frequencies and longer lifetimes, (9a) and (9b) are even smaller quantities.

* Received September 30, 1963.

[1] B. R. Nag and P. Das, "Microwave propagation in semiconductors with carrier density varying in time," IRE Trans. on Microwave Theory and Techniques, vol. MTT-10, pp. 564–567; November, 1962.

[2] H. Jacobs, F. A. Brand, J. D. Meindl, S. Weitz, R. Benjamin and D. A. Holmes, "New microwave techniques in surface recombination and lifetime studies," Proc. IEEE, vol. 51, pp. 581–592; April, 1963.

[3] S. Deb and B. R. Nag, "Measurement of lifetime of carriers in semiconductors through microwave reflection," *J. Appl. Phys.*, vol. 33, p. 1604; April, 1962.

[4] The typographical error in the final term of (11) of Nag and Das[1] has been corrected in (7) of this communication.

[5] R. A. Smith, "Semiconductors," The Syndics of the Cambridge University Press, London, England, pp. 217–218; 1961.

Using (9), (7) can be written as

$$i_{\alpha_2} = -\frac{\omega^2\mu\epsilon_1 + j\omega\mu\sigma_1}{2\gamma_s^2}$$

and (6) becomes

$$\Gamma = \gamma_s\left(1 - \frac{\omega^2\mu\epsilon_1 + j\omega\mu\sigma_1}{2\gamma_s^2}\cdot e^{-t/\tau_c}\right), \quad (10)$$

which is identical to (5).

For longer lifetimes and higher frequencies, the approximations used above are even better. In very short lifetime materials, such as Gallium Arsenide, the correction terms included in the analysis of Nag and Das[1] may become important. However, with short lifetime materials, the diffusion length becomes quite small and it is difficult to satisfy the assumption of uniform carrier density throughout the semiconductor.

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Letter to the Editor*

The following paper is felt to be of interest to workers in the microwave measurements field:

B. E. Rabinovich, "Method free from mismatching errors for measuring the loss of attenuators," *Izmeritel'naya Tekhnika*, no. 3, pp. 44–47; March, 1962.

The English translation appears in *Measurement Techniques*, no. 3, pp. 238–243; 1962.

Rabinovich places one directional coupler ahead and one after the device under test. The change in the ratio of incident power of both side arms is related to the insertion loss. The effects of finite directivity and main line VSWR are considered. By the use of auxiliary phaseshifters or sliding loads, the maximum and minimum influence of these effects can be determined.

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A Simple Relation Between Cavity Q and Maximum Rejection for Narrow-Band Microwave Band-Stop Filters*

In the course of designing band reject filters, one must often determine the unloaded cavity $Q(Q_u)$ required to achieve a predetermined maximum rejection. This situation is analogous to that of determining the Q_u required to give a specific midband loss (due to dissipation) in bandpass filters.

The design of microwave band reject filters possessing narrow stop bands has been given detailed consideration by Young, Matthaei and Jones.[1] The following relation is developed in this communication based on transformation from the low-pass filter prototype:[2]

$$L_{max} = 20\sum_{i=1}^{n}\log_{10}\omega_1' w g_i Q_{ui} + 10\log\frac{g_0 g_{n+1}}{4} \quad (1)$$

where

L_{max} = the maximum rejection in db
w = the normalized bandwidth at half-power (Butterworth) or equiripple (Chebyshev) points
Q_{ui} = the unloaded Q of the ith cavity
n = the number of cavities
ω_1' = the normalized band edge radian frequency of the low-pass prototype
g's = the low-pass prototype element values.

If the use of equal Q_u cavities is assumed, as is most often true, (1) can be written as

$$L_{max} = 20n\log_{10}(\omega_1' w Q_u) + 20\sum_{i=1}^{n}\log_{10} g_i + 10\log_{10}\left(\frac{g_0 g_{n+1}}{4}\right). \quad (2)$$

Now ω_1' is unity by definition and w is simply the cavity decrement, the reciprocal of the loaded $Q(Q_L)$. Thus, (2) becomes

$$L_{max} = 20n\log_{10}\frac{Q_u}{Q_L} + 20\sum_{i=1}^{n}\log_{10} g_i + 10\log_{10}\left(\frac{g_0 g_{n+1}}{4}\right). \quad (3)$$

For the situation where the rejection response follows the *maximally flat* (Butterworth) characteristic, g_0 and g_{n+1} are unity. Then, the third term of (3) is −6.02 db. For n equal to 1 through 10, the second term of (3) varies between +6.00 and +6.02 db. Therefore, a good approximation for the relation of L_{max} to Q_u is

$$L_{max} = 20n\log_{10}\left(\frac{Q_u}{Q_L}\right)\text{ db} \qquad n = 1, 2, 3, \cdots, 10. \quad (4)$$

The analogous relation for midband dissipation loss in bandpass filters is[3]

$$L_{diss} = 20n\log\frac{Q_u}{Q_u - Q_L}. \quad (5)$$

A graph of (4) is shown in Fig. 1. Curves of (5) have been widely published.[4]

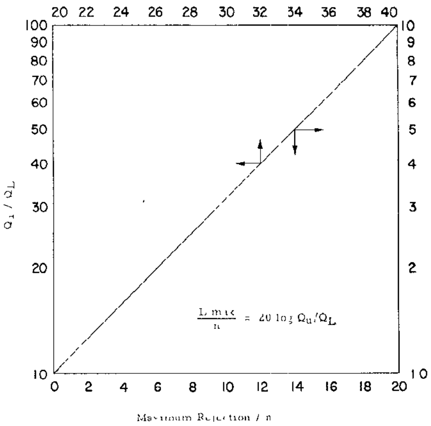

Fig. 1—Unloaded Q vs maximum rejection for Butterworth band reject filters.

Verification of (4) has been carried out by means of a simple test. A pair of UHF filters, a band reject filter and a bandpass filter, were constructed using identical resonators. The bandpass filter was a two-resonator unit. It had a 0.9-db insertion loss at f_0=253 Mc and a 3-db pass band of 6.7 Mc. The band reject filter was a three-resonator structure which exhibited approximately 105-db maximum rejection (measured as 35 db per resonator) at f_0=254 Mc with a 3-db bandwidth of 20 Mc.

Applying (4) to the band reject data, Q_L=254/20=12.7 and $JL/3$=35 db. Thus, Q_u/Q_L=57 and Q_u=724.

Eq. (5) was used in the bandpass case. From the two-resonator bandpass data, Q_L=253/6.7=37.86, $JL/2$=4.5, Q_L/Q_u=0.0505 and, therefore, Q_u=749. Thus, the simple experiment has shown satisfactory agreement between (4) and the widely accepted bandpass Q (5).

The range of n [in (4)] for a Butterworth response can be extended beyond 10 if one can show that the second term of (3) is approximately +6.0 db for arbitrary n. For the Butterworth response, g_i is given by

$$g_i = 2\sin\left[\frac{(2i-1)\pi}{2n}\right]. \quad (6)$$

To satisfy the 6.0 db requirement, it must be shown that

$$\prod_{i=1}^{n}\sin\left[\frac{(2i-1)\pi}{2n}\right] = \frac{1}{2^{n-1}}. \quad (7)$$

This has been done numerically for the range of n from 1 to 10. Extension to higher values of n can be made by repeated numerical solutions of (7) using a computer. An analytic approach is to prove the identity of (7). This has been done and the result is tabulated as series No. 1049 by Jolley,[5] where n is any positive integer. On the basis of this result, (4) can be used for any value of n.

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* Received October 11, 1963.

[1] L. Young, G. L. Matthaei, and E. M. T. Jones, "Microwave band-stop filters with narrow stop bands," IRE Trans. on Microwave Theory and Techniques, vol. 10, pp. 416–427; November, 1962.
[2] S. B. Cohn, "Direct-coupled resonator filters," Proc. IRE, vol. 45, pp. 187–196; February, 1957.
[3] "Very High Frequency Techniques," vol. II, McGraw-Hill Book Co., New York, N. Y., p. 745; 1947.
[4] For example, "The Microwave Engineer's Handbook," 2nd ed., p. T-99; 1963.
[5] L. B. W. Jolley, *Summation of Series*, 2nd. ed. rev., Dover Publications, p. 194; 1961.